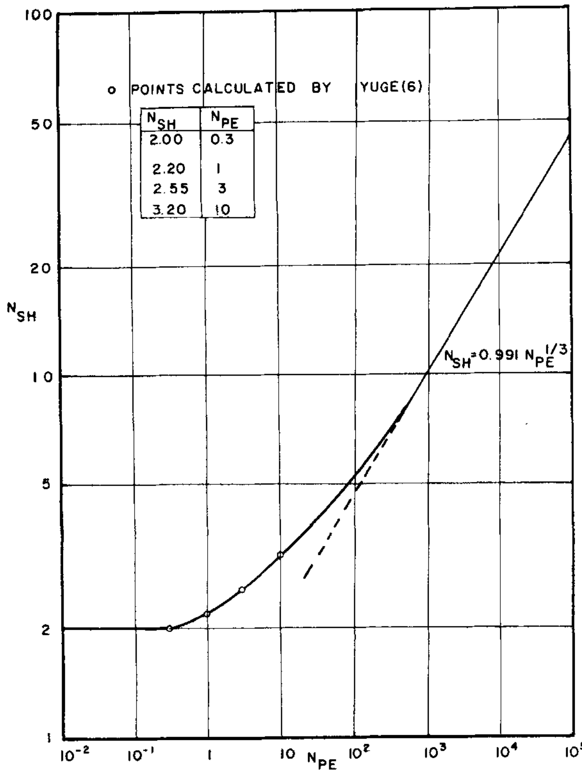

Fig. 1. Transport to spheres in Stokes flow.

$$N_{Sh} = \frac{(3\pi)^{2/3} \qquad N_{Pe}^{1/3}}{4(2)^{1/3}\ \Gamma(4/3)}$$
$$= 0.991\, N_{Pe}^{1/3} \qquad (7)$$

Levich finds for the numerical value of the constant 0.997 (3). The complete N_{Sh} vs. N_{Pe} curve is shown in Figure 1 based on the values calculated by Yuge and Equation (7). The thin boundary-layer approximation is probably satisfactory for $N_{Pe} > 10^2$, but it would be desirable to have more numerical calculations for the range $10^3 > N_{Pe} > 10$. The Sherwood number is very close to 2 for $N_{Pe} < 0.3$. It should be noted that the approximate curve given in (*1*) falls quite close to Figure 1.

ACKNOWLEDGMENT

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NOTATION

a = radius of sphere (or cylinder), cm.

c_1 = ratio of point concentration to surface concentration, dimensionless

D = diffusion coefficient, sq.cm./sec.

k = mass transfer coefficient, cm./sec.

N_{Pe} = Péclet number, $2aU/D$, dimensionless

N_{Re} = Reynolds number, $2aU/\nu$, dimensionless

N_{Sc} = Schmidt number, ν_{Nu}/D, dimensionless

N_{Sh} = Sherwood number, $2ak/D$, dimensionless

u_1 = ratio of point velocity in x direction to U, dimensionless

U = fluid velocity at infinity, cm./sec.

v_1 = ratio of point velocity in y direction to U, dimensionless

x_1 = ratio of distance measured along a meridian from the stagnation point to a, dimensionless

y_1 = ratio of distance normal to surface to a, dimensionless

Greek Letters

α = arbitrary constant, dimensionless

η = boundary layer variable, dimensionless

ν = kinematic viscosity, sq.cm./sec.

primed quantities denote dummy variables

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Mass Transfer in Stirred Vessels

R. B. OLNEY

Shell Development Company, Emeryville, California

Treybal (*11*) has analyzed certain liquid-liquid mass transfer data obtained in baffled vessels equipped with flat-blade turbines (*3, 9*). For his model he used the Calderbank-Korchinski (*2*) approximation for diffusion in the drop phase, while the coefficient for the outside liquid film was estimated from the equation of Johnson and Huang (*5*) for dissolution of solid rings fixed in place in a baffled, stirred vessel. Other over-all transfer data (*9*) were obtained with the system kerosene-water-butylamine (solute) with marine propellers and spiral turbines in a 15-in. diameter baffled vessel, but the results were not included in Treybal's analysis. It seems worthwhile therefore to show a similar analysis of these data.

One will use a still simpler idealization in which one neglects the continuous phase resistance and assumes that the over-all transfer rate is limited solely by molecular diffusion in the drops. One assumes spherical drops, so that $a = 6\phi/d$. Further one assumes that the Kolmogorov (*8*) or Hinze (*4*) mechanism for maximum stable drop size in an isotropic homogeneous turbulence may be used to estimate the mean drop size in stirred vessels, with suitable adjustment of the dimensionless constant:

$$d = C_1 \left(\frac{\sigma g_c}{\rho_c} \right)^{0.6} \epsilon^{-0.4} \quad (1)$$

For stirred vessels the power/mass is determined directly from experiments, or one may take

$$\epsilon = \frac{4}{\pi} C_p \frac{N^3 L^5}{HD^2} \quad (2)$$

$$C_p = \frac{P\, g_c}{\rho\, N^3 L^5} \quad (3)$$

The measurements of drop size or interfacial area reported by Vermeulen, Williams, and Langlois (*12*), Calderbank (*1*), Kafarov and Babanov (*7*), and others indicate that as a first approximation Equation (1) gives the mean drop size in stirred vessels. In general the data from these three investigations show that C_1 is of the order of 0.1 for the systems studied (no mass transfer). Some effects of ϕ and of L or L/D on C_1 are demonstrated (*12, 7*), but one will neglect these in the first analysis. Independently some observations of drop size made during mass transfer studies with the kerosene-butylamine-water system in another stirred device indicate that C_1 for this system is two to three times that for systems having similar properties but with no transfer occurring. Therefore one will take $C_1 \cong 0.25$ in the analysis of the propeller and spiral turbine data.

A drop-side coefficient $k_D a$ is defined in terms of the Newman equation:

$$e^{-k_D a t/\phi} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-4\pi^2 n^2 \mathcal{D} t/d^2} \quad (4)$$

and the experimental coefficients $K_k a$, based on the kerosene (feed) phase, are calculated from

$$K_k a = \frac{Q_k (y_1 - y_0)}{V(y_0 - x_0/m)} \quad (5)$$

Here d is obtained from Equation (1). Values of ϕ and ϵ were taken directly from the experiments (*9*), and the following properties were used: $\rho_c = 1.0$ g./cc., $\sigma = 25$ dyne/cm., and $\mathcal{D} = 1.2 \times 10^{-5}$ sq. cm./sec., where $\mathcal{D}$ is estimated (*13*) with a molecular weight of 180 for the kerosene phase assumed.

The ratio $(K_k a/k_D a)$ is given as a function of the power input in Figure 1 for marine propellers of 4-, 6-, 8-, and 10-in. diameter and for spiral turbines of 4-, 6-, and 8.5-in. diameter, with those runs in which ϕ and P were determined used. The 8-in. propeller data include runs at three impeller submergences, three residence times, and three kerosene/water flow ratios, and the 6-in. turbine data include runs at two impeller submergences. The data scatter fairly uniformly around a ratio $K_k a/k_D a = 1.0$, and the average deviation from this ratio is 26% for the propellers and 32% for the spiral turbines. Some effect of impeller size remains, which may be the result of an effect of L on K_k or an effect of L or L/D on C_1.

The flat-blade turbine data from this same investigation (*9*) that were analyzed by Treybal are not given on the figure. They show about the same scatter as for the propellers and spiral turbines, but they give an average value of 2.3 for $K_k a/k_D a$, with $C_1 = 0.25$; this agrees with the effective diffusivity, 2.25 $\mathcal{D}$, used by Treybal (*11*). Various speculations can be made as to why the flat-turbine data do not agree with those for the other two impeller types.

The results are not entirely conclusive in view of the limitations of the analysis and of the data. Nevertheless the implication is clear that for some drop systems motion within the drop is largely suppressed despite the turbulent motions outside the drop. Similar findings have of course been shown more conclusively for simpler transfer environments; one recent example is the single-drop study of Johnson and Hamielec (*6*).

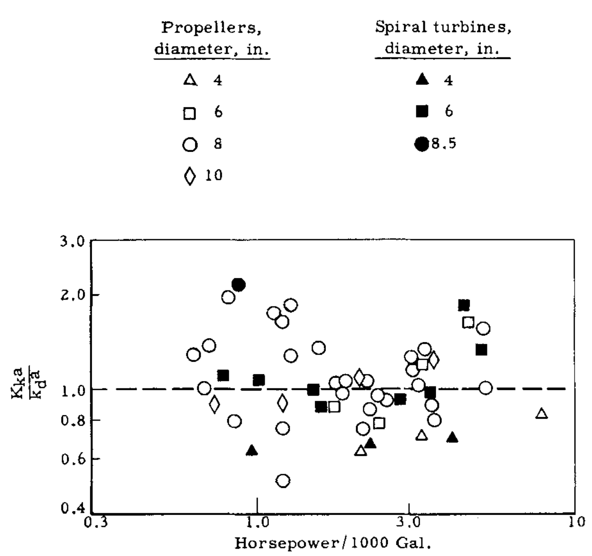

Fig. 1. Mass transfer coefficients, kerosene-water-butylamine, marine propellers, and spiral turbines in 15-in. diameter baffled vessel (9).

NOTATION

C_1 = dimensionless constant
C_p = power coefficient for geometrically similar impellers in baffled vessels (*10*)
D = vessel diameter
d = mean drop size
$\mathcal{D}$ = molecular diffusivity of solute in drop phase
H = liquid depth
L = impeller diameter
m = distribution coefficient
Q_k = flow rate of feed phase
t = $V\phi/Q_k$ = average drop residence time
V = vessel volume
x_0, y_1, y_0 = solute concentrations in the outlet water and inlet and outlet kerosene phases, respectively
ϵ = power/mass
ϕ = measured volume fraction of dispersed phase in vessel

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